Study of Matrix Inversion in MATLAB and Cramer's Rule on Electrical Circuits

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Abstrak

Memahami prinsip-prinsip fisika, terutama dalam rangkaian listrik 2-loop, memerlukan penerapan Hukum Kirchhoff untuk membentuk sistem persamaan linear. Meskipun metode substitusi-eliminasi umumnya digunakan untuk menyelesaikan sistem ini, metode tersebut menjadi tidak efisien ketika menangani lebih dari tiga variabel. Sebagai alternatif, Aturan Cramer, yang melibatkan penggunaan determinan matriks dan metode kofaktor, terbukti lebih efektif, terutama untuk sistem yang lebih besar. Penelitian menunjukkan bahwa Aturan Cramer dan metode inversi matriks menghasilkan hasil yang konsisten untuk perhitungan arus. Nilai arus yang diperoleh dari rangkaian pertama dan kedua cocok dengan yang dihitung menggunakan metode kofaktor. Teknik-teknik ini tidak terbatas pada rangkaian listrik; teknik-teknik ini juga dapat diterapkan di bidang teknik lainnya, seperti analisis sistem dinamis dan mekanika. Pemahaman yang mendalam tentang Aturan Cramer, kofaktor, dan inversi matriks sangat penting untuk menganalisis sistem fisika dan teknik. Pada rangkaian pertama, hasilnya adalah i_1 = 0,25 A, i_2 = 2,5 A, dan i_3 = 2,75 A, sedangkan pada rangkaian kedua, hasilnya adalah i_1 = 1 A, i_2 = 2 A, dan i_3 = 1 A.

Abstract

Understanding the principles of physics, especially in a 2-loop electrical circuit, requires applying Kirchhoff's Laws to form a system of linear equations. While substitution-elimination methods are commonly used to solve these systems, they become inefficient when dealing with more than three variables. As an alternative, Cramer's Rule, which involves using matrix determinants and the cofactor method, proves to be more effective, particularly for larger systems. The research shows that both Cramer's Rule and the matrix inversion method produce consistent results for current calculations. The current values obtained from the first and second circuits match those calculated using the cofactor method. These techniques are not limited to electrical circuits; they can also be applied in other engineering fields, such as dynamic systems analysis and mechanics. A solid understanding of Cramer's Rule, cofactors, and matrix inversion is essential for analyzing both physical and engineering systems. In the first circuit, the results are i_1 = 0.25 A, i_2 = 2.5 A, and i_3 = 2.75 A, while in the second circuit, the results are i_1 = 1 A, i_2 = 2 A, and i_3 = 1 A.

Keywords : matlab, matrix inversion, electrical circuit

Introduction

Electrical engineering [1] is a field that requires a deep understanding of concepts, logical thinking, and strong mathematical skills. To study this discipline, it's essential to have a structured grasp of these concepts to apply them effectively. Electrical engineering [2] explores natural phenomena, often expressed in mathematical equations, to describe the relationships between various physical phenomena. These equations capture connections between both macroscopic and microscopic phenomena, as well as the fundamental laws of the field. This underscores the importance of mathematics in helping us understand electrical engineering [3] comprehensively.

However, many people still find electrical engineering [4] difficult. This is because the field requires more than just memorization; it demands a deep understanding of concepts and the ability to solve complex problems. One area that often causes confusion is the analysis of 2-loop electrical circuits [5], which is a key topic in dynamic electrical circuits [6]. Many students and instructors find 2-loop circuits challenging, mainly because fundamental concepts like electric current [7] and poten-

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tial difference are not always well understood. Misconceptions about the relationship between current [8] and voltage in dynamic circuits further complicate the learning process. These misunderstandings can lead to errors when applying essential principles like Kirchhoff's Laws, which are crucial for analyzing electrical circuits[9]. Another challenge is understanding the differences between direct current (DC) [10] nd alternating current (AC) circuits[11]. DC circuits[12], for example, have distinct characteristics regarding voltage measurement and the effect of resistance on current [13] flow. Many people struggle to translate theoretical knowledge into practical calculations, whether for simple or complex circuits [14]. This difficulty can hinder deeper understanding of electrical circuits [15].

Solving electrical circuits [16] typically involves using Kirchhoff's Laws[17], which consist of two main principles: Kirchhoff's First Law (the current law) [18] and Kirchhoff's Second Law (the voltage law) [19]. Kirchhoff's First Law states that the sum of currents [20] entering and leaving a junction in a circuit [21] must be zero. In other words, the current [22] flowing into a point in a circuit [23] must be equal to the current [24] flowing out of that point, as no current [25] is lost or created at that junction. This principle is also known as the conservation of charge. Kirchhoff's Second Law, on the other hand, states that the sum of electromotive forces (EMFs) [26] and the voltage drops around a closed loop must be zero.

Electromotive forces (EMFs) [27] provide the energy that drives current [28] through a circuit [29], while voltage drops occur due to resistive elements like resistors. According to Kirchhoff's Voltage Law (KVL), the total voltage supplied by the energy sources must balance the voltage lost across resistive components in a closed loop. By applying both Kirchhoff's Current Law (KCL) and KVL, we obtain a system of linear equations [30] (SLE) that can be solved to find the current [31] and voltage in the circuit[32]. These equations describe the relationships between current [33] and voltage in the circuit , allowing us to determine the current [34] in each branch and the voltage across each component. For example, in a circuit [35] with multiple resistors and EMF sources, Kirchhoff's Laws help form equations [36] that are then solved to calculate the current in each branch. This approach is especially useful for analyzing more complex circuits, such as parallel or series circuits [37], or circuits [38] with multiple voltage sources and resistances. Kirchhoff's Laws offer a systematic and effective method to solve a wide range of electrical circuit [39] analysis problems.

To solve these systems, several methods can be used, such as Gauss-Jordan elimination, Cramer's Rule, and substitution. However, when dealing with systems that involve more than two variables, substitution can become time-consuming and prone to errors. As a result, more efficient methods are often preferred. The Gauss-Jordan and Gauss-Seidel methods are commonly used to solve systems of linear equations, particularly when multiple iterations are required. Cramer's Rule, however, is especially useful for larger systems of equations with more variables. This method transforms the system of equations into a matrix multiplication form: $[P][X] = [Q]$, where [P] is the coefficient matrix, [X] is the column matrix containing the variables (such as current), and [Q] is the constant matrix. Cramer's Rule offers a direct solution using matrix determinants, which can be computed using various techniques like the cofactor method.

Using Cramer's Rule makes solving systems of linear equations faster and more accurate compared to traditional methods. This efficiency makes it a valuable tool for analyzing complex electrical circuits, especially those involving dynamic electrical materials. These mathematical techniques help researchers and engineers solve problems more effectively. The determinant of a matrix is a key value derived from a square matrix and plays an important role in solving linear systems, analyzing stability, and even in areas like graph theory and geometry. Various methods are used to calculate determinants depending on the matrix size and the complexity of the problem. For 3x3 matrices, Sarrus' rule is a common method, but it is limited to that specific size. A more versatile approach is the cofactor method, which works for matrices of any size, including n x n matrices. This method involves calculating a cofactor for each element based on its position. While systematic, it can be more computationally intensive than other techniques. Another widely used approach is Gaussian elimination, which transforms the matrix into an upper or lower triangular form. The determinant is then calculated by multiplying the diagonal elements. This method is efficient for larger matrices but may face numerical stability issues when dealing with matrices that have very large or very small values. Decomposition methods, which break the matrix into simpler triangular matrices, are also commonly used. Each method has its strengths, and the choice depends on the matrix size and the specific problem at hand.

For larger matrices, the cofactor method is often preferred despite the extra steps required. In this research, Cramer's Rule will be applied as an alternative approach to solve electrical circuits [40], specifically focusing on two-loop circuits [41]. The cofactor method will be used to calculate the necessary determinants, and matrix inversion will be integrated into the analysis using MATLAB[42], offering a more efficient and accurate solution.

MATLAB [43] will be utilized to simplify matrix inversion calculations, improving both speed and accuracy. MATLAB's powerful built-in functions for numerical analysis automatically compute determinants and matrix inversions, reducing the risk of errors that may arise from manual calculations. This study aims to demonstrate how Cramer's Rule can be effectively used to analyze two-loop electrical circuits [44] and highlight the advantages of MATLAB [45] in solving engineering problems. Furthermore, this research seeks to contribute to the development of technology-based education in electrical engineering [46] by showcasing the practical application of mathematical theories.

Ultimately, the goal of this research is to demonstrate Cramer's Rule as an effective and efficient solution for analyzing DC two-loop electrical circuits. Additionally, the study aims to expand the understanding of how MATLAB [47] can be used to solve engineering problems, particularly in electrical circuit analysis, and contribute to advancing educational methods in the field of electrical engineering.

This research is based on Ohm's Law, a fundamental principle in electrical theory discovered by Georg Simon Ohm in 1827. Ohm's Law is crucial for understanding how electrical circuits work, as it describes the relationship between three key components: voltage (V), current (I), and resistance (R). According to this law, the current flowing through a conductor is directly proportional to the voltage applied across it and inversely proportional to the resistance. This relationship is expressed by the simple equation:

$$V = I\ X\ R \tag{1}$$

Where:

- V is the voltage across the component (measured in volts, V),
- I is the current flowing through the component (measured in amperes, A),
- R is the resistance of the component (measured in ohms, Ω).

Ohm's Law is an essential tool for engineers and scientists because it helps predict how changes in voltage, current, or resistance will affect a circuit's behavior. For example, if the resistance in a circuit increases, the current will decrease, assuming the voltage remains constant. On the other hand, if the voltage increases, the current will rise, assuming the resistance stays the same. This understanding enables engineers to design and optimize circuits, ensuring that electrical systems operate safely and efficiently.

Although Ohm's Law is often applied to simple resistive circuits, its principles are fundamental for analyzing more complex electrical systems. It helps guide the design of components such as resistors, capacitors, inductors, and transistors. Ohm's Law also plays a crucial role in the operation of electric power systems, telecommunications, and modern consumer electronics. Technologies like electric vehicles, renewable energy systems, and integrated circuits all rely on precise control and measurement of current flow, which is made possible through effective electrical management based on Ohm's Law.

Ohm's Law also helps engineers troubleshoot electrical issues by calculating voltage drops, current flows, and power consumption in various parts of a circuit [48]. This capability makes it easier to identify faulty components or optimize circuit efficiency.

In summary, Ohm's Law is more than just a historical discovery, it's a practical and indispensable tool in electrical engineering [49]. Its applications span a wide range of industries and continue to drive the development of modern technologies. A solid understanding and application of Ohm's Law are essential for anyone involved in the design, maintenance, or innovation of electrical systems.

In simpler terms, Ohm's Law explains that the voltage (V) applied to a conductor is directly proportional to the current (I) that flows through it, as long as the resistance (R) remains unchanged. Essentially, this means that if you increase the voltage, more current will flow, but only if the resistance doesn't change. For example, turning up the voltage in a circuit will cause more electrons to move, which increases the current. On the other hand, if the voltage is held constant, increasing the resistance (such as adding a resistor to the circuit) will reduce the current because the electrical flow is "slowed down" by the higher resistance.

The relationship is captured by the equation V=I×RV, which helps engineers and scientists design and analyze electrical circuits [50]. This simple yet powerful equation is fundamental to understanding how electricity behaves in all kinds of circuits. Whether it's a basic household device like a light bulb or electric heater, or a sophisticated system like integrated circuits in modern electronics, Ohm's Law applies everywhere. It allows engineers to predict how changes in voltage or resistance will affect the current, making it easier to design efficient circuits and troubleshoot electrical problems. For instance, knowing how much current will flow through a component helps engineers prevent overloading circuits or ensure that devices operate within safe limits.

Ohm's Law is crucial in many applications. In household appliances, such as toasters or water heaters, it ensures proper functioning by regulating electrical heating. In more complex systems like smartphones, computers, or electric vehicles, Ohm's Law is essential for controlling the current and voltage to ensure reliable operation. From power distribution networks to miniature gadgets, Ohm's Law remains a fundamental tool for understanding and managing electrical flow. However, Ohm's Law is more than just a formula. The resistance (R) of a conductor can change depending on environmental factors, such as temperature. For example, in metals like copper and aluminum, resistance increases as temperature rises, which must be accounted for in circuits operating in high-heat environments. Similarly, in semiconductors, the relationship between voltage and current isn't always linear, meaning Ohm's Law doesn't apply the same way as it does for regular conductors.

In engineering and electronics, Ohm's Law is vital because it allows engineers to predict and control circuit behavior. By understanding the relationship between voltage, current, and resistance, they can calculate the precise voltage needed to achieve a specific current or determine the resistance required to prevent excessive current flow. This ensures that circuits work as intended and that components are protected from damage. For example, in designing mobile phones, computers, and household appliances, engineers use Ohm's Law to optimize performance, selecting appropriate resistors, capacitors, and other components to ensure devices function efficiently, consume minimal power, and operate reliably under varying conditions.

In power systems, Ohm's Law ensures the safe and efficient distribution of electricity. Engineers use it to design power grids that deliver the correct voltage from generators to consumers while minimizing losses due to resistance in transmission lines. By calculating appropriate voltage levels and resistance characteristics, they ensure power is distributed reliably over long distances with minimal waste. Ohm's Law is also key in designing transformers and circuit protection devices, which help maintain the integrity of the entire power distribution system.

Overall, Ohm's Law is the foundation of modern electrical engineering and electronics. It spans industries from consumer electronics to large-scale power generation, helping engineers design more reliable, efficient, and safe systems that meet the demands of today's technology-driven world.

Ohm's Law is not only fundamental for understanding current, voltage, and resistance, but it also plays a crucial role in understanding electrical power (P). Electrical power is the rate at which energy is consumed or produced in a circuit, and it is directly related to voltage and current. The power dissipated by a component can be calculated using the formula $P=V\times I$, where P is power in watts (W), V is the voltage across the component, and I is the current flowing through it. This equation helps determine how much energy is being converted into heat or light, which is especially important when designing circuits to ensure safe operation. Additionally, Ohm's Law allows us to expand the power equation in different forms depending on the circuit conditions. For instance, if the resistance R is known, power can also be calculated as $P=I2\times R$, or alternatively, $P=V2/R$. These variations are useful when either the current or the voltage is known, but the other is not directly measurable. They are especially helpful for calculating the power dissipated by resistive components, like resistors, where energy is typically lost as heat. By applying Ohm's Law and these power equations, engineers can ensure that components in a circuit are properly rated for their power dissipation, preventing overheating or damage. For example, in high-power applications such as electric motors or power supplies, it's crucial to select components that can safely handle the expected power levels to avoid failures or fires. In smaller devices, such as smartphones or laptops, efficient power management is key for extending battery life and preventing overheating during use. In summary, Ohm's Law helps engineers understand and control current, voltage, and resistance, and it also aids in calculating electrical power. By using the expanded power equations, engineers can design circuits that are efficient and safe, ensuring components can handle the required power without risk of damage or overheating. This is critical in various applications, from everyday electronics to large-scale industrial systems, where reliable power management is essential for performance and safety.

Overall, Ohm's Law is not just a theoretical principle but a practical and indispensable tool that plays a critical role in both everyday life and the advancement of modern technology. Its simplicity and reliability make it a foundational concept in electrical engineering [51], helping to bridge the gap between abstract theory and real-world applications. A solid understanding of Ohm's Law enables engineers, technicians, and designers to create electrical systems that are both efficient and safe, ensuring optimal performance and functionality.

By applying Ohm's Law, we can predict how electrical current will behave in various circuits, whether it's a small household appliance or a complex industrial system. For example, it allows us to calculate the exact voltage required to drive a certain current through a circuit [52] or determine the appropriate resistance to limit the flow of current to safe levels. This capability is crucial for avoiding overloading, reducing the risk of overheating, and ensuring the longevity of electronic components. In consumer electronics, such as smartphones, laptops, and wearable devices, Ohm's Law helps optimize the performance and energy efficiency of circuits, contributing to longer battery life and stable operation.

Methodology

This research explores how linear algebra can be used to solve engineering problems, specifically focusing on analyzing DC circuits with two loops. The main objective of the study is to develop an efficient and accurate method for calculating the electric currents in these circuits [53], using linear algebra techniques, with particular emphasis on Cramer's rule. Cramer's rule is a well-established method for solving systems of linear equations, and in this context, it is applied to find the current values in each loop of a circuit based on Kirchhoff's laws. Kirchhoff's First Law governs current distribution at each node, while Kirchhoff's Second Law relates the voltage around each loop. These laws give rise to three linear equations representing the currents at different points in the circuit: I1, I2, and I3.

After establishing these equations, the research converts them into matrix form, which simplifies the process of solving the system of equations. While Cramer's rule is used for this purpose, the study also explores the matrix inverse method as an alternative approach. This method is particularly helpful for larger, more complex systems, as it allows for direct calculation of current values without the need for time-consuming and error-prone determinant and cofactor calculations. Software tools like MATLAB [54] are especially useful in this process, as they enhance both the speed and accuracy of the calculations, particularly when dealing with 3x3 matrices, which are the focus of this study.

MATLAB [55] offers several advantages, especially in terms of computational speed and precision. It can handle complex numerical calculations quickly and accurately, making it an invaluable tool for analyzing more complicated circuits. Additionally, MATLAB [56] provides visualization tools that help users interpret and analyze the results more easily. The software not only speeds up the solution of the system of equations but also reduces the likelihood of human error, which is common in manual calculations.

This study enhances computational efficiency by using the matrix inverse method, particularly when dealing with systems involving multiple variables. This approach allows for direct calculation of current values, streamlining the process compared to other methods. Additionally, the matrix inverse method can be extended to handle more complex circuits, such as AC circuits or those with non-linear components, which require more advanced mathematical techniques.

In conclusion, this research demonstrates that applying linear algebra, particularly Cramer's rule and the matrix inverse method, provides an efficient and effective way to solve engineering problems in circuit analysis. These methods are not only useful for analyzing two-loop DC circuits, but they can also be applied to more complex electrical engineering challenges, such as circuits with multiple loops or non-linear elements. Moreover, the use of MATLAB [57] significantly accelerates the calculation process, improving both the accuracy and efficiency of circuit design and analysis. The techniques developed in this study are also adaptable to other engineering fields, making them valuable tools for modern electrical engineering applications.

Result and Discussion

By applying Kirchhoff's laws to a two-loop circuit with two voltage sources and three resistors, we can derive linear equations that describe the relationship between the currents in each loop. For example, let's define the current in the first loop as I1 and the current in the second loop as I2. By analyzing the voltage changes across the components, we can create two equations that reflect Kirchhoff's Voltage Law (KVL) for each loop.

For the first loop, the equation would be: I1R1+I2R2=V1, where R1 and R2 are are the resistances, and V1 is the voltage source in the first loop. For the second loop, the equation is:

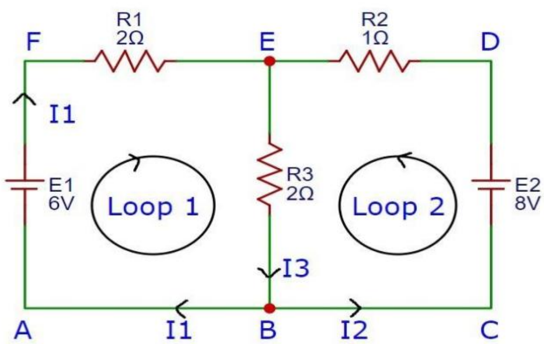
I1R2+I2R3=V2, where R3 is the resistance in the second loop, and V2 is the voltage source in the second loop. After obtaining these two equations, we can solve them algebraically to find the values of the currents I1 dan I2. If we add a third equation that describes the relationship between the currents at another point in the circuit, we then have a system of three equations with three variables to solve. In this case, we can use Cramer's Rule with matrix determinants to find the solution. Below is the calculation for two electrical circuit problems.

Example 1: From the first circuit (Fig. 1.), we have the following equations:

Loop I : 2I1 + 2I2 = 6

Loop II : 1I 2 + 2I3 = 8

Hukum I Kirchoff : I1 + I 2 = I 3.



Gambar 1: *Electrical Circuit 1*

Steps:

1. Transform the above equations into a system of linear equations:

$2x_1 + 0x_2 + 2x_3 = 6$

$0x_1 + 1x + 2x = 8$

$$1x_1 + 1x_2 - 1x_3 = 0 \qquad (2)$$

2. Convert the system into matrix multiplication form.

3. Use Cramer's rule to determine $x_1 = I_1$, $x_2 = I_2$, dan $x_3 = I_3$.

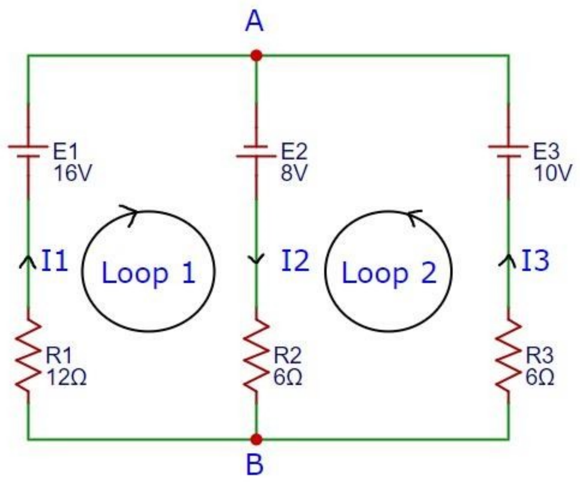
4. Determine the determinants of matrices P, P1, P2, and P3 using the minor-cofactor method.

5 . After calculation, the results are $I_1 = 0.25$ A, $I_2 = 2.5$ A, and $I_3 = 2.75$ A.

Example 2: For the second circuit (Fig. 2.), we have the following equations:

Loop I : 12 I1 + 6I2 = 24

Loop II : 6 I2 + 6I3 = 18



Gambar 2: *Electrical Circuit 2*

Steps:

1. Transform the above equations into a system of linear equations:

$12x_1 + 6x_2 + 0x_3 = 24$

$0x_1 + 6x_2 + 6x_3 = 18$

$$1x_1 + 1x_2 - 1x_3 = 0 \tag{3}$$

2.Convert the system into matrix multiplication form.

3.Use Cramer's rule to determine $x_1 = I_1$, $x_2 = I_2$, dan $x_3 = I_3$.

4.Determine the determinants of matrices P, P1, P2, and P3 using the minor-cofactor method.

5.After calculation, the results are $I_1 = 1A$, $I_2 = 2A$, and $I_3 = 1A$.

In a DC two-loop electrical circuit, differences in the direction of the voltage sources between the two loops can cause a sign reversal in the current calculation. This happens because the assumed direction of current in each loop might not align with the actual polarity of the voltage sources. For example, if a loop is traversed from the negative (-) terminal to the positive (+) terminal of a voltage source, the current calculated using methods like Cramer's Rule will flow in the opposite direction compared to the conventional direction of the voltage source, which flows from positive (+) to negative (-). This difference in direction has a significant impact on the calculation of the electromotive force (EMF) for that loop. The EMF, which represents the potential difference, depends on how the voltage source is traversed within the loop. If the current flows against the polarity of the voltage source (from negative to positive), the voltage contribution is negative in the loop's equation. Conversely, if the current flows in the same direction as the voltage source, the EMF is positive. These directional differences affect the signs in the equations derived from Kirchhoff's Voltage Law (KVL), which are used to calculate the currents and voltages across each component. It's important to accurately account for these sign changes to ensure that the current and voltage calculations accurately reflect the true behavior of the circuit.

The directionality of voltage sources and current flow is crucial to ensuring the consistency of the solution when using Cramer's Rule. Since this method solves a system of linear equations, the sign of the current depends on how the voltage sources are treated relative to the chosen loop directions. Misinterpreting the direction of the voltage sources can lead to incorrect current directions or voltage drops across components, ultimately affecting the accuracy of the circuit's performance analysis. Therefore, it is essential to carefully consider the orientation of voltage sources and current directions when solving for electrical parameters in a two-loop DC circuit. Additionally, the direction of the electromotive force (EMF) is directly linked to the flow of electrons in the circuit, which is a key concept in electrical engineering and physics. According to standard conventions, electrons flow from the negative terminal to the positive terminal of a voltage source. This movement creates the electrical current in the circuit, which, by convention, flows from positive to negative. The direction of current flow is tied to the motion of electrons, and understanding this relationship is vital for accurate circuit analysis. If the loop direction in the circuit opposes the orientation of the voltage source, the behavior of the electrons will be affected. In this case, the voltage source will try to push the electrons in the opposite direction of the assumed current direction. As a result, the actual electron flow will reverse, and the current calculated using methods like Cramer's Rule will reflect this reversal. Specifically, both the magnitude and direction of the current will be reversed. For example, if the current was initially calculated to flow in one direction, the reversal could indicate that the actual current flows in the opposite direction, which would change the expected voltage drops and power dissipation across components. This change in direction is significant for the circuit's analysis and understanding. When solving circuit equations with Cramer's Rule or any other method, it's crucial to account for such reversals to ensure that all variables, such as current and voltage, are correctly oriented. Incorrectly assuming the direction of current can lead to errors in calculating electrical parameters like voltage or power consumption, which would affect the accuracy of the circuit's performance analysis. Therefore, the interaction between the EMF direction, electron flow, and current direction is a critical factor for solving electrical circuits correctly.

.This code performs the matrix inversion and multiplication to calculate the solution for the unknowns in Circuit A.= = When applying Cramer's Rule with the minor-cofactor method in circuit analysis, it's crucial to pay careful attention to the choice of loop direction. The loop direction should align with the direction of the electromotive force (EMF). The loop direction represents the assumed current flow around a closed circuit loop, while the EMF is the force driving the current, determined by the polarity of the voltage sources in the circuit. If the loop direction is incorrectly aligned with the voltage source, the resulting current calculations can be reversed. This misalignment can cause discrepancies between theoretical results and the actual behavior of the circuit. Such errors become more problematic when analyzing complex circuits, leading to incorrect predictions for the current and voltage values across components. Additionally, mistakes in determining the loop direction or assigning the correct signs to the voltage sources can significantly affect the accuracy of the calculations. For example, it could result in a positive current being calculated when the actual current should be negative, or vice versa. This issue is particularly pronounced in multi-loop circuits, where multiple voltage sources and components interact, further complicating the analysis. To prevent these errors, it's important to carefully apply Kirchhoff's Voltage Law (KVL) for each loop, ensuring that the voltage contributions from each component are accounted for consistently with the assumed current direction. A solid understanding of current flow and EMF directionality is essential for correctly applying Cramer's Rule. In summary, having a clear understanding of the relationship between loop direction, voltage sources, and current direction is key when using Cramer's Rule in circuit analysis, especially for complex circuits. By ensuring these directions are consistently defined and followed, the analysis will be more accurate, leading to more reliable results and better circuit design, optimization, and troubleshooting.

The Matrix Inversion Method is a powerful technique for solving systems of linear equations, commonly used in circuit analysis to determine unknown currents or voltages. For Circuit A, after applying the matrix inversion method, the solution obtained is x=[0.2500;2.5000;2.7500]. This solution represents the values of the unknown variables, which could correspond to currents or voltages in different parts of the circuit.To solve for the unknowns, the following MATLAB code was used:

```
Matlab Copy
a = [2 0 2; 0 1 2; 1 1 -1];
b = [6; 8; 0];
x = inv(a) * b;
```

This code performs the matrix inversion and multiplication to calculate the solution for the unknowns in Circuit A.

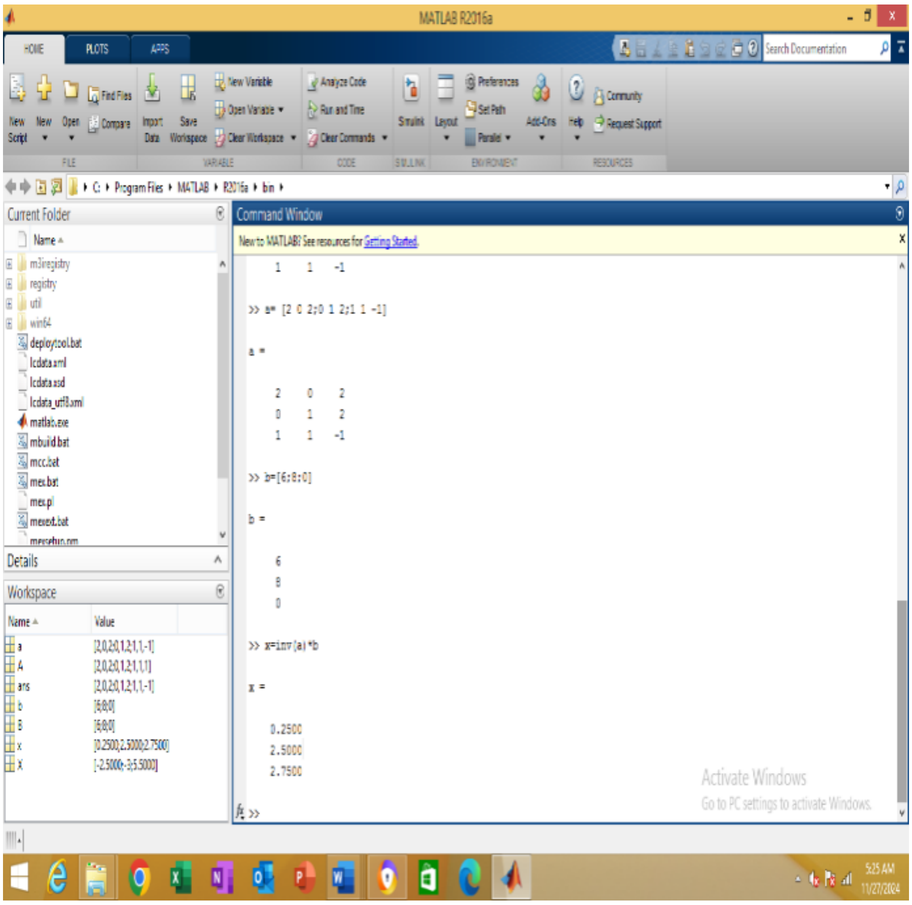
In this case, A represents the coefficient matrix of the system, and b is the vector of constants from the equations. By calculating the inverse of matrix A and multiplying it by b, we obtain the solution vector x, which provides the values for the unknowns. For example, the result x=[0.2500;2.5000;2.7500] indicates the values of the currents or voltages in different loops or components of the circuit. Similarly, for Circuit B, applying the same matrix inversion method gives the solution x=[1.0000;2.0000;1.0000], calculated as follows:

```
Matlab Copy
a = [12 6 0; 0 6 6; 1 -1 1];
b = [24; 18; 0];
x = inv(a) * b;
```

This method efficiently provides the solution for Circuit B as well.

In the case of Circuit B, A represents the coefficient matrix and b is the constant vector. The MATLAB function inv(A) calculates the inverse of the coefficient matrix, and by multiplying this inverse with b, we obtain the solution vector x. This solution, x=[1.0000;2.0000;1.0000], provides the values for the unknowns, such as currents or voltages, at various points in Circuit B. Both circuits demonstrate how the matrix inversion method can efficiently solve the systems of linear equations that arise in electrical circuit analysis. This approach is especially helpful when dealing with multiple equations and unknowns. With MATLAB's built-in matrix manipulation functions, this method is easy to implement, even for larger and more complex systems. The results obtained give the necessary values to analyze and optimize the performance of the circuit, showcasing the method's reliability and precision in electrical engineering applications. Figure 3. shows the results from the MATLAB matrix inversion calculation.

For Circuit A, Cramer's Rule also gives the same solution: x=[0.2500;2.5000;2.7500]. Similarly, for Circuit B, the solution is x=[1.0000;2.0000;1.0000]. While Cramer's Rule produces correct results in both cases, it is more suitable for smaller systems of linear equations, where the number of variables and equations is limited. This is because Cramer's Rule involves calculating the determinants of the coefficient matrix and modified matrices, which are obtained by replacing certain columns with the constant vector. This process becomes inefficient and time-consuming as the size of the system increases.



Gambar 3: *Results from MATLAB-based matrix inversion*

This study compares the use of matrix inversion in MATLAB and matrix inversion through Cramer's Rule for solving electrical circuit problems. Both methods are used to solve the systems of linear equations that arise in circuit analysis, but they differ significantly in their approach and application.

1. Matrix Inversion Using MATLAB:
   - Approach: In MATLAB, matrix inversion is performed using built-in functions like inv(). If we have a system of linear equations in the form Ax=b, we can compute the solution x directly using the formula x=A−1.b.
   - Advantages:
     - Efficiency: MATLAB can quickly and efficiently solve matrix inversion problems, especially for large matrices. It uses optimized numerical algorithms that minimize the risk of errors, even with unstable matrix inversions.
     - Ease of Use: MATLAB makes the process easy and fast by utilizing pre-existing functions, requiring minimal effort from the user.
     - Accuracy: MATLAB is reliable for handling complex, large-scale numerical calculations and accounts for rounding errors, ensuring higher precision.
   - Usage in Electrical Circuits: In circuit analysis, this method is often used to solve the systems of equations derived from Kirchhoff's current and voltage laws. Matrix inversion allows for direct calculation of current and voltage values in complex circuits.

2. Matrix Inversion Using Cramer's Rule:
   - Approach: Cramer's Rule solves systems of linear equations using determinants. For a system Ax=b, each variable xi can be found by calculating xi=det(Ai)/det(A), where Ai is the matrix obtained by replacing the i-th column of matrix AAA with the constant vector b.
   - Advantages:
     - Mathematical Insight: Cramer's Rule provides a deeper understanding of the properties of linear systems and determinants, making it more intuitive for theoretical analysis.

  - Suitable for Small Matrices: When the matrix A is small (e.g., 2x2 or 3x3), Cramer's Rule is quick and easy to apply.
- Limitations:
  - Inefficient for Large Matrices: For large matrices, calculating determinants and forming the Ai matrices for each variable becomes highly inefficient, time-consuming, and resource-intensive.
  - Dependence on Determinants: If the determinant of matrix A is zero, the system has no unique solution, and Cramer's Rule cannot be applied.
- Usage in Electrical Circuits: Cramer's Rule is more suitable for simpler circuits with fewer equations and variables. For more complex circuits, it becomes impractical, and matrix inversion methods are recommended instead.

Tabel 1: Comparison Between the Two Methods

Aspect	Matrix Inversion (MATLAB)	Cramer's Rule
Method	Uses numerical algorithms to invert the matrix.	Uses determinants and replacement matrices.
Advantages	Fast, efficient, and accurate for large matrices.	Easy to understand, intuitive, and suitable for small matrices.
Limitations	Can experience numerical errors for nearly singular matrices.	Inefficient for large matrices and cannot be used if the determinant is zero.
Usage in Electrical Circuits	Ideal for analyzing circuits with large and complex linear systems.	Suitable for simpler circuits with fewer equations.

In summary, while Cramer's Rule is useful for small, simple circuits, MATLAB's matrix inversion is far more efficient and reliable for larger, more complex electrical circuit problems.

Conclusion

MATLAB is ideal for analyzing complex electrical circuits, especially those involving many linear equations and variables. It excels at handling complex numerical calculations quickly and accurately, particularly when dealing with large matrices. In circuits analyzed using Kirchhoff's laws, or circuits with multiple components like resistors, capacitors, and inductors, we often encounter systems of equations that need to be solved for various currents and voltages. In such cases, matrix inversion allows us to directly solve these systems. MATLAB's built-in functions, like inv(), enable us to compute the inverse of a circuit's coefficient matrix and multiply it by the constant vector to find the solution. This approach is far more efficient than manual methods, such as Cramer's Rule, which becomes impractical for large systems due to the complexity of calculating determinants. Additionally, MATLAB handles complex numerical problems with high precision and stability, which is essential for analyzing large and dynamic circuits. Another advantage of MATLAB is its data visualization tools, which allow us to graphically display currents, voltages, and other circuit elements, making it easier to interpret the circuit's behavior. In summary, MATLAB saves time, reduces the risk of calculation errors, and provides an efficient way to analyze even the most complex electrical circuits with many components and variables. Cramer's Rule, on the other hand, is better suited for simpler circuits with only a few linear equations and variables. In these cases, the system of equations can be solved by hand, and Cramer's Rule provides an intuitive way to find the solution. For example, in a circuit with just a few resistors and voltage sources, solving a small number of equations is manageable. However, as the circuit becomes more complex with more variables, Cramer's Rule becomes inefficient. Calculating the determinant for each variable becomes increasingly difficult and time-consuming. Manual calculations are also more prone to errors, especially when forming replacement matrices. Additionally, as the size of the system grows, the method requires more memory and computational power, making it impractical for larger systems. In short, while Cramer's Rule is helpful for understanding the theory behind linear systems and determinants, it is not ideal for complex electrical circuits. For such cases, methods like matrix inversion in MATLAB are much more efficient and accurate, minimizing errors and handling larger systems with ease. Acknowledgements The author would like to express gratitude to the rectorate and the foundation for their support, which has contributed to the realization of this research.

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